THE AMERICAN MATHEMATICAL MONTHLY.

DEVOTED TO THE
SOLUTION OF PROBLEMS IN PURE AND APPLIED MATHEMATICS, PAPERS ON MATHEMATICAL SUBJECTS, BIOGRAPHIES OF NOTED MATHEMATICIANS, ETC.

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BIOGRAPHY.

ALEXANDER MACFARLANE, M. A., D. Sc., LL. D.

BY J. M. COLAW.

ALEXANDER MACFARLANE was born at Blairgowrie, Scotland, April 21st, 1851. He was educated at the public school, and at 13 became a regular pupil teacher in the employment of the Education Department.

In 1869, having finished his apprenticeship as a teacher and saved a little money, Mr. Macfarlane went straight to the University of Edinburgh. At that time the curriculum for Master of Arts consisted of three departments, — classical, mathematical, and philosophical; and it was customary for the more ambitious students to take the degree with honors in one of these departments. Mr. Macfarlane first entered the Junior classes in Latin and in Greek, and at the end of the session stood fourth in the former and fifth in the latter, in classes of 200, largely composed of High School graduates. He perceived that to carry himself through college it was necessary either to sacrifice a large part of his time to teaching, or else to study hard and pay his way by means of money prizes. He chose the bolder alternative. At the beginning of his second year he won in open competition the Miller scholarship, worth $400. At the end of that year he stood very high in Senior Latin and Greek and in Junior Mathematics. At the beginning of the third year he won in open competition the Spence scholarship, worth $1,000. The financial difficulty was now solved; there remained a choice of a department for honors. He was urged by the professor of Latin to go forward in the Classics, but he felt that there was more scope for originality in philosophy. In his third year he studied Senior Mathematics, Natural Philosophy and Logic. It was the custom of Professor Kelland to introduce Quaternions to his senior students. The addition of vectors

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was intelligible, but the product of vectors seemed to be a universal difficulty. The professor explained that in *i j* the left-hand vector was to be considered as a sort of corkscrew turning the right-hand vector through a right angle; but he did not explain how in *i i* it ceased to be a corkscrew. To get light on the subject Mr. Macfarlane bought a copy of Tait's *Treatise on Quaternions*, but found that it was addressed to mathematicians.

Before he entered the class of Logic Mr. Macfarlane was familiar with the works of Hamilton and Mill, and when a member of the class he read, at the invitation of the professor, a paper which criticised the statement of the law of Excluded Middle given by Jevons in his *Lessons on Logic*. It was his intention to study for honors in Logic and Philosophy, but perceiving how much they depended on the principles of science, and especially of exact science, he took up the advanced classes in Mathematics and Physics as a secondary study. In Experimental and Mathematical Physics he gained the highest honors and the personal friendship of Professor Tait, then, as now, the greatest figure in the University. In 1874 he was appointed Neil Arnott instructor in Physics, and in 1875 finished an unusually extensive course of undergraduate study by taking the degree of M. A. with honors in Mathematics and Physics. The University record showed that he had passed each of the seven subjects of the pass examinations with high distinction. Having, after graduation, won in a competitive examination the Maclaren fellowship, worth $1,500, he proceeded to study for the recently instituted degree of Doctor of Science. After one year spent on Chemistry, Botany, and Natural History, and two years on Mathematics and Physics, he obtained the doctorate in 1878. His thesis was an experimental research on the conditions governing the electric spark, and it was subsequently published in the *Transactions of the Royal Society of Edinburgh*. It also brought him under the notice of the celebrated electrician and philosopher, Clark Maxwell, who made various suggestions for its extension.

In 1878 Dr. Macfarlane was elected a Fellow of the Royal Society of Edinbeugh, and the first contribution which he read personally was a memoir on the Algebra of Logic. The memoir was referred by the Council to the professors of mathematics and of logic, and they reported that it was too mathematical for the one and too logical for the other to enable them to say what its value was. Dr Macfarlane enlarged the memoir and published it as a small volume under the title of *Principles of the Algebra of Logic* (1879). The volume was received with favor, and brought the author into correspondence with Munro, Jevons, Venn, Cayley, Harley, Schroeder and Halsted, who was then lecturing on the mathematical logicians at Johns Hopkins University. The main idea propounded is that of a limited and definite universe; also Euler's diagrams were further developed. In 1879 he attended the meeting of the British Association at Sheffield, and there met many of the British savants.

During 1880 Dr. Macfarlane was interim Professor of Physics at the University of St. Andrews, and in 1881 he was appointed for the usual period of three years Examiner in Mathematics in the University of Edinburgh. During these years he contributed to the Royal Society of Edinburgh a series of

experimental papers on electricity, and a series of mathematical papers on the *Analysis of the Relationships of Consanguinuity and Affinity*. A paper on this subject, which he read before the Anthropological Institute of London, contains as perfect a notation for relationship as is the Arabic notation for numbers. These papers, as well as those on the Algebra of Logic, now form part of the history of Exact Logic. He also contributed to the Royal Society of Edinburgh a *Note on Plane Algebra*, which stated briefly the view he had arrived at concerning the imaginary algebra of the plane. It states that the fundamental quantity is versor rather than a vector, a view in advance of Argand's, and indeed of much that has been written more recently. By means of this algebra of the plane he deduced many series, some of which he propounded as problems in the *Educational Times* and the *Mathematical Visitor*. It was also during his tenure of office as examiner that he prepared the volume on *Physical Arithmetic*, a pioneer work, whose express object is to elucidate the logical processes involved in the application of arithmetic to physical problems.

In 1885 Dr. Macfarlane was called to the chair of physics at the University of Texas, where he became a colleague of his fellow logician, Dr. Halsted. That same year he met many of the American savants at the Ann Arbor meeting of the American Association. In 1887 he received the honorary degree of LL. D. from the University of Michigan on the occasion of their semi-centennial. His first years at the University of Texas were wholly taken up with organizing the department, but in 1889 he published as a sequel to *Physical Arithmetic* a volume of *Elementary Mathematical Tables*, distinguished for their comprehensiveness and uniformity. In 1889 he visited Paris at the time of the Exposition and met many of the continental savants at the meeting of the French Association.

On his return from Europe, he began to publish the results of his study of the algebra of space, which he approached as a logical generalization of the Algebra of the Plane. These papers are as follows: 1° *Principles of the Algebra of Physics*, read before the Washington meeting of the American Association in 1891, states the fundamental difficulties in the theory of Quaternions, lays stress on the distinction between vectors and versors, and deals mostly with the products of vectors. 2° *On the Imaginary of Algebra*, read at the Rochester meeting in 1892, gives an historical and critical account of the different interpretations of $\sqrt{(-1)}$, takes up the functions of versors, and shows that there are at least two distinct geometrical meanings of $\sqrt{(-1)}$. 3°. *The Fundamental Theorems of Analysis Generalized for Space*, contributed to the New York Mathematical Society in 1892, investigates and proves the generalized form of the Binomial and other theorems, and thus establishes the principles of spherical trigonometrical analysis. 4°. *On the Definitions of the Trigonometric Functions*, read before the Mathematical Congress at Chicago in 1893, defines these functions so as to apply to the circle, hyperbole, ellipse, logarithmic spiral, and a complex curve partly circular, partly hyperbolic. 5°. *The Principles of Elliptic and Hyperbolic Analysis*, read at the same place and time, extends spherical trigometrical analysis to the other surface of the second

order. 6°. *The analytical treatment of alternating currents*, read before the International Electrical Congress at the same time, shows that plane algebra is the analysis needed for the problems of alternating currents 7°. *On physical addition or composition*, read before the Madison meeting of the American Association in 1893, treats in a uniform manner of the composition of various physical quantities located in space, ending with the composition of screw-motions. 8°. *On the fundamental principles of exact analysis*, read before the Philosophical Society of Washington in 1894, discusses the fundamental laws of algebra, and the logical principle of generalization in analysis. 9°. *The principles of differentiation in space analysis*, recently read before the American Mathematical Society at New York, investigates the differentiation of versors, and publishes the true generalization of Taylor's theorem for space.

In 1891 Dr. Macfarlane took an active part in organizing the Texas Academy of Science, and for two years acted as its Honorary Secretary. He contributed many papers, among which may be mentioned "An Account of the Rainmaking Experiments in San Antonio," an article describing and criticising the various modern methods of rainmaking, and a paper on "Exact Analysis as the Basis of Language," where his knowledge both of languages and of mathematics comes into play.

In 1894 Professor Macfarlane resigned from the University of Texas. Throughout the nine years he labored there, he gave the new University the full benefit of his varied experience as a teacher, his accurate knowledge of University affairs, and his widespread reputation as a savant. The course in mathematical physics was so well developed as to call forth a special article in the *Rievista di Matematica*, published at Turin, Italy.

Professor Macfarlane, in addition to being a member of numerous American and British societies, is a corresponding member of the *Sociedad Cientifica Antonio Alzate*, of Mexico, and the *Circolo Matematico di Palermo*, Italy. Personally he is a characteristic Scotsman, sturdy, persevering, with a relish for hard work, thoughtful, courageous in his convictions, and endowed with more than the average share of the *perfervidum ingenium Scotorum*. He is unmarried, but it is announced that in this, as in other matters, good fortune awaits him. And as he is still a young man, it is not likely that we have seen the last of his contributions to mathematical analysis.

To the editors of the *Electrical World* we are indebted for the loan of the electrotype.

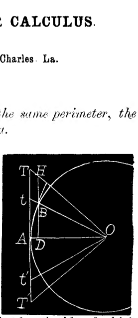
ISOPERIMETRY WITHOUT CURVES OR CALCULUS.

By PROFESSOR P. H. PHILBRICK, M. Sc., C. E., Lake Charles. La.

[Continued from the November Number.]

PROPOSITION IX. *If two regular polygons have the same perimeter, the one having the greater number of sides has the greatest area.*

Describe a circle with any radius $AO=r$ and circumference $2\pi r=c$.



Take AB and AC respectively the mth and nth part of a semi-circumference, and draw the secants OBt and OCT to meet the tangent AtT. Draw also DBH parallel to AT, and the tangent Be. Then, At is one-half of one side of a regular polygon of m sides, whose apothem is AO; and AT is one-half of one side of a regular polygon of n sides, whose apothem is likewise AO.

Let P=the perimeter of the polygon, the length of each side of which is $2AT$, and p=the perimeter of the polygon, the length of each side of which is $2At$.

Let a=the arc AB and A=the arc AG.

$$\text{Then, } P=AT\frac{c}{A},\ \ p=At\ \frac{c}{a},\ \therefore\ \frac{P}{p}=\frac{AT}{At}\cdot\frac{a}{A}.$$

Now, $BH > Be >$ arc BC and $BD <$ arc AB.

$$\text{Dividing gives, } \frac{BH}{BD} > \frac{\text{arc } BC}{\text{arc } AB} \text{ or } \frac{DH}{DB} > \frac{\text{arc } AC}{\text{arc } AB} = \frac{A}{a}.$$

But $\frac{AT}{At}=\frac{DH}{DB}$ and therefore $\frac{AT}{At} > \frac{A}{a}$.

$$\text{Multiplying by } \frac{a}{A} \text{ we have, } \frac{AT}{At}\cdot\frac{a}{A} > 1.$$

Therefore, $\frac{P}{p} > 1$ or $P > p$.

Hence, for the same apothem, the perimeter of the polygon of the greater number of sides is the smaller.

Therefore, for *equal* perimeters, the apothem of the polygon of the greater number of sides must be the greater; and since, for equal perimeters, the areas vary as the apothem, the area of the polygon having the greater number of sides is likewise the greater.

PROPOSITION X. *If two regular polygons have the same area, the one having the greater number of sides has the least perimeter.*

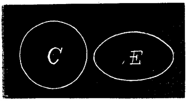
If the perimeters were equal, then (Prop. IX) the area of the one hav-

ing the greater number of sides would be the greater. Hence, since the areas are equal, the perimeters of the polygon having the greater number of sides is the smaller.

PROPOSITION XI. *Of all isoperemetric figures, the circle has the maximum area.*

We prove by (Prop. VIII) that of regular isoperemetric polygons, that having the greatest number of sides has the greatest area, and hence if the number of sides of any regular polygon be continually increased, keeping its perimeter the same, its area will be continually increased; and as the circle is the limiting figure in conformity to which the regular polygon continually approaches, as the number of its sides is made greater and greater, the circle is that figure, which for a given perimeter contains the maximum area.

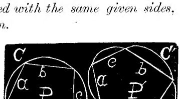
PROPOSITION XII. *Of all plane figures containing the same area, the circle has the minimum perimeter.*



Let C be a circle and E any other figure having the same area as C.

Now, by Prop. XI, if the perimeter of C was equal to that of E its area would be greater than that of E; but since it is the same, its perimeter must be less than that of E.

PROPOSITION XIII. *Of all polygons formed with the same given sides, that which can be inscribed in a circle is a maximum.*



Let the polygon P, having the sides a, b, c, d, and e, be inscribed in a circle, and the polygon P' formed with the same sides, not be inscriptible.

Upon the sides a, b, c, etc., of the polygon P' construct circular segments, equal to those standing upon the corresponding sides of the polygon P.

Then the whole figure C' thus found, has the same perimeter as the circle C.

Hence, (Prop. XI) area of $C >$ area of C'; and subtracting the circular segments from both, we have, $P > P'$.

[Concluded.]

THE INSCRIPTION OF REGULAR POLYGONS.

By LEONARD E. DICKSON, M. A., Fellow in Pure Mathematics, University of Chicago.

CHAPTER V.

[Continued from the December Number.]

II. *When the number of sides is a multiple of* 5.

In the regular 25-*gon*, $A_5-A_{10}=1$, being chords of the regular pentagon. But $A_1-A_2+A_3-A_4+A_5-\ldots\ldots-A_{12}=1$.

$\therefore\quad (A_1-A_4-A_6+A_9+A_{11})+(-A_2+A_3+A_7-A_8-A_{12})=0.$

The product of these two groups expanded is seen to be 0. Hence, each group equals zero.

The sum of the chords $A_1, -A_4, -A_6, A_9, A_{11}$ equals 0; the sum of their 10 products two at a time$=2(A_2-A_3-A_7+A_8+A_{12})+5(A_{10}-A_5)$ $=-5$; the sum of their 10 products three at a time $=-6(A_1-A_4-A_6+A_9+A_{11})+2(A_2-A_3-A_7+A_8+A_{12})=0$; the sum of their 5 products four at a time$=5(A^2-A_5)=5$; the product of all five $=2(A_1-A_4-A_6+A_9+A_{11})-(A_2-A_3-A_7+A_8+A_{12})+A_5=A_5$. Hence, they are the roots of $x^5-5x^3+5x-A_5=0$.

Similarly, $A_2, -A_3, -A_7, A_8, A_{12}$ are the roots of $x^5-5x^3+5x-A_{10}=0$.

In the regular 35-*gon*, $A_7-A_{14}=1$; $A_5-A_{10}+A_{15}=1$.

$\therefore\quad (A_2-A_9+A_{12}+A_{16})+(A_6+A_8-A_1-A_{13})+(A_4-A_3-A_{11}-A_{17})=1.$

Write A, B, C for these three groups respectively.

Then $A+B+C=1$; $AB=3B+3C+4(A_{10}-A_{15})$;

$$AC=3A+3B+4(-A_5-A_{15});\ BC=3A+3C+4(A_{10}-A_5).$$

$\therefore\quad AB+AC+BC=6(A+B+C)+8(-A_5+A_{10}-A_{15})=-2.$

$$ABC=C\{3-3A+4(A_{10}-A_{15})\}=3C-3AC+4C(A_{10}-A_{15}),$$

expanded, $=-1$.

Hence, A, B, C are the roots of $x^3-x^2-2x+1=0$. But (Chapter I.) $A_5, -A_{10}, A_{15}$ are the roots of this cubic. By inspection, or by a table of natural cosines, we determine which of the roots in the two sets correspond; viz, $A=A_5$; $B=A_{15}$; $C=-A_{10}$.

$$\therefore\quad \begin{cases} A_1-A_6-A_8+A_{13}+A_{15}=0 \\ A_2-A_5-A_9+A_{12}+A_{16}=0 \\ A_3-A_4-A_{10}+A_{11}+A_{17}=0. \end{cases}$$

We may prove by our usual method that:

$A_1, -A_6, -A_8, A_{13}, A_{15}$ are the roots of $x^5-5x^3+5x-A_5=0$

$A_2, -A_5, -A_9, A_{12}, A_{16}$ are the roots of $x^5-5x^3+5x-A_{10}=0$

$A_3, -A_4, -A_{10}, A_{11}, A_{17}$ are the roots of $x^5-5x^3+5x-A_{15}=0$.

By induction, for a regular polygon of $n=5m$ sides:

$$A_1-A_{m-1}-A_{m+1}+A_{2m-1}+A_{2m+1}=0.$$

$$A_2-A_{m-2}-A_{m+2}+A_{2m-2}+A_{2m+2}=0.$$

Generally, $A_s-A_{m-s}-A_{m+s}+A_{2m-s}+A_{2m+s}=0.$

We may prove this trigonometrically by use of the formula for the sum of two cosines; but more elegantly thus:

Since $A_m-A_{2m}=1$, being chords of the regular pentagon,

$$A_s=A_s(A_m-A_{2m})=A_{m-s}+A_{m+s}-A_{2m-s}-A_{2m+s}.$$

The sum of the chords, $A_s, -A_{m-s}, -A_{m+s}, A_{2m-s}, A_{2m+s}$ equals 0; the sum of their products two at a time

$$=2(A_{2s}-A_{m-2s}-A_{m+2s}+A_{2m-2s}+A_{2m+2s})+5(A_{2m}-A_m)=-5;$$

the sum of their products three at a time$=0$; four at a time$=5$; the product of all five$=A_{5s}$. Hence, they are the five roots of $x^5-5x^3+5x-A_{5s}=0$, where s is any integer $\overline{<}\ \frac{m-1}{2}$; and $A_5, -A_{10}, A_{15}, \ldots \pm A_{5s}, \ldots$ are the $\frac{m-1}{2}$ roots of the equation (4) for the regular m-gon.

If m is not divisible by 5, one chord of every group of 5 chords is a root of this equation (4). For one and only one of the subscripts s, $m-s$, $m+s$, $2m-s$, $2m+s$ is always divisible by 5, as is seen by replacing some by their equivalents: $5m-s$, $m-s$, $4m-s$, $2m-s$, $3m-s$. The remaining four chords will be determined by a series of quadratics whose co-efficients are linear functions of the roots of (4).

Thus, if s be divisible by $5, \pm A_s, \pm A_{2s}, \pm A_{3s}$, are roots of (4).

Then $(-A_{m-s}-A_{m+s})+(A_{2m-s}+A_{2m+s})=-A_s$; $(-A_{m-s}-A_{m+s})(A_{2m-s}+A_{2m+s})=2(-A_m+A_{2m})+(-A_{m-2s}-A_{m+2s}+A_{2m-2s}+A_{2m+2s})=-2-A_{2s}$. $\therefore$ $(-A_{m-s}-A_{m+s})$ and $(A_{2m-s}+A_{2m+s})$ are the roots of the quadratic $x^2+A_sx-(2+A_{2s})=0$.

Now $A_{m-s} \cdot A_{m+s}=A_{2m}+A_{2s}$; $A_{2m-s} \cdot A_{2m+s}=A_{2s}-A_m$.

The sum and product of each pair of chords being known in terms of the roots of (4), it follows that, if the $\frac{m-1}{2}$ chords of the regular m-gon be found, we can find all the chords of the regular $5m$-gon by solving a series of quadratics.

However, if m be divisible by 5, the five chords in any of the above groups are all, or not one of them, roots of equation (4) for the regular m-gon; for the subscripts s, $m-s$, $m+s$, $2m-s$, $2m+s$, are either all or not one of them divisible by 5, according as s is or is not divisible by 5. Hence, we can not avoid or lower the above quintic.

The regular 5-m gon depends for inscription upon the same equations

as does the regular m-gon, if m be prime to 5; but also upon one or more quintics of the above form, if m contains the factor 5.

III. *When the number of sides is divisible by 7.*

For a regular polygon of $n=7m$ sides:

$$A_s-A_{m-s}-A_{m+s}+A_{2m-s}+A_{2m+s}-A_{3m-s}-A_{3m-s}=0.$$

For, $A_m-A_{2m}+A_{3m}=1$, being chords of the regular 7-gon.

Hence, $A_s=A_s(A_m-A_{2m}+A_{3m})=A_{m-s}+A_{m+s}-A_{2m-s}-A_{2m+s}+A_{3m-s}+A_{3m+s}$.

By the usual method of proof, $A_s, -A_{m-s}, -A_{m+s}, A_{2m-s}, A_{2m+s}, -A_{3m-s}, -A_{3m+s}$ are the 7 roots of $x^7-7x^5+14x^3-7x-A_{7s}=0$, where $s \overset{=}{>} \frac{m-2}{2}$, and $A_7, -A_{14}, A_{21}, \ldots \pm A_{7s}, \ldots$ are the roots of the equation (4) for the regular m-gon.

If m is prime to 7, one and only one chord of each of the above groups of 7 chords is a root of this equation (4).

The remaining six chords will be determined by a cubic and three quadratics, whose coefficients are linear functions of the roots of (4).

Thus, if s be divisible by 7, $\pm A_s, \pm A_{2s}, \pm A_{3s}, \ldots$ are roots of (4).

Write A for $=A_{m-s}-A_{m-s}$, B for $A_{2m-s}+A_{2m+s}$, C for $-A_{3m-s}-A_{3m+s}$.

Then $A+B+C=-A_s$; $AB+AC+BC=-4(A_m-A_{2m}+A_{3m})$
$+2(-A_{m-2s}-A_{m+2s}+A_{2m-2s}+A_{2m+2s}-A_{3m-2s}-A_{3m+2s})=-4-2A_{2s}$.

ABC expanded gives $3A_s+A_{3s}$. Hence, A, B, C are the roots of
$x^3+A_sx^2-(4+2A_{2s})x-(3A_s+A_{3s})=0$.

But $A_{m-s} \cdot A_{m+s}=A_{2m}+A_{2s}$, etc. Hence, if the $\frac{m-1}{2}$ chords of the regular m-gon be found, we can find all the chords of the regular $7m$-gon by solving a cubic and 3 quadratics.

However, if m be divisible by 7, the 7 chords in any of the above groups are all or none of them roots of equation (4) for the regular m-gon. Hence, we can neither lower the above septics nor avoid them.

The regular 7m-gon depends for inscription upon the same equation as the regular m-gon, together with an additional cubic, if m be prime to 7; but together with one or more additional septics, if m contains the factor 7.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton), Ph. D., (Johns Hopkins), Member of the London Mathematical Society, and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the December Number.]

SCHOLION I. Here it is permitted to observe a notable difference from the hypothesis of acute angle.

For in this the general concurrence of straights cannot be demonstrated in this way, as often as any straight falling upon two, makes two internal angles toward the same parts less than two right angles; cannot, I say, be directly demonstrated, even if in this hypothesis the aforesaid general concurrence be admitted, as often as one of the two angles is right.

For although the straight AD be perpendicular even to the straight AP; in which case it certainly could not concur with another perpendicular PL (Eu. I. 17.); nevertheless the two angles together DAX, PXA, could be less than two right angles, in accordance with the aforesaid hypothesis, since in it the two angles together PAX, PXA may be less (P. IX.) than one right angle.

But it was worth while to have observed this. Just as, infact, solely from the admission of this general concurrence when one of the angles is right, and with an assigned incident however small, the hypothesis of acute angle can be demolished; this we will show after the three next propositions.

[To be continued.]

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

33. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

A wine-merchant's *apparent* profit is 25% of his sales which are 10% of cost less water. What is his actual rate per cent. of profit?

I. Solution by P. S. BERG, Apple Creek, Ohio, and the PROPOSER.

The *apparent* selling-price is $\frac{125}{100}$ of the cost. On account of the

costless water, the *actual* selling-price is ($\frac{10}{9}$ of $\frac{125}{100}$) of $100\% = 138\frac{8}{9}\%$. Hence, the actual rate per cent. of profit is $38\frac{8}{9}\%$.

II. **Solution by FRANK HORN, Columbia, Missouri, and Professor H. J. GAERTNER, Wilmington College, Wilmington, Ohio.**

II.
1. 100% = apparent value.
2. 125% = selling price of apparent value.
3. $90\% = 100\% - 10\%$ = value of quantity sold for 125%.
4. $\therefore$ $\frac{125}{90}\%$ = what 1% sells for.
5. $138\frac{8}{9}\% = 100 \times \frac{125}{90}\%$ = real selling price.
6. 100% = true value.
7. $38\frac{8}{9}\% = 138\frac{8}{9}\% - 100\%$ = rate of gain.

$\therefore$ The actual rate of gain is $38\frac{8}{9}\%$.

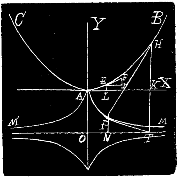
34. A chain 100m long, weighing 14 oz. to the foot, is suspended from points on a level 80m apart. What is the sag, the batter at the ends, and the horizontal tension? [From *Wentworth & Hill's High School Arithmetic*.]

Solution by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Kidder Institute, Kidder, Missouri

The form of the chain fulfilling the conditions of the problem is the curve known as the *catenary*. Let B and C be the points of suspension of the chain, E any point in the chain, $AL = x$, $EL = y$.

Let $AE = s$ and $w = 45.93$oz. the weight of a metre of length of the chain.

Then ws = the weight of the portion AE = the load suspended at E, or the vertical tension at E. Let aw = the horizontal tension at A, the weight of a units of length. Let EF be a tangent at E; then if EF represents the tension at E, EI and IF will represent the horizontal and vertical tensions respectively, at E.



Hence, $\dfrac{dy}{dx} = \dfrac{FI}{EI} = \dfrac{ws}{wa} = \dfrac{s}{a} \ldots (1)$. But $ds = \sqrt{(dy^2 + dx^2)}$. $\therefore$ $dy = \sqrt{(ds^2 - dx^2)}$.

$s \div a = \sqrt{(ds^2 - dx^2)} \div dx$, whence

$$\frac{dx}{ds} = \frac{a}{\sqrt{(a^2+s^2)}}. \quad \therefore \quad x = a\int \frac{ds}{\sqrt{(a^2+s^2)}}$$

$= a \log_e (s + \sqrt{a^2+s^2}) + c$. Since $x=0$, when $s=0$, $c = -a \log a$.

$\therefore$ $x = a \log_e [(s + \sqrt{a^2+s^2}) \div a] \ldots (2)$. From (2), we have

$s = \dfrac{a}{2}(e^{\frac{x}{a}} - e^{-\frac{x}{a}}) \ldots (3)$. From (1) and (3) $\dfrac{dy}{dx} = \frac{1}{2}(e^{\frac{x}{a}} - e^{-\frac{x}{a}})$.

$\therefore$ $y = \dfrac{a}{2}(e^{\frac{x}{a}} - e^{-\frac{x}{a}}) + c$. Since $y=0$ when $x=0$, $c = -a$.

$\therefore$ $y = \dfrac{a}{2}(e^{\frac{x}{a}} - e^{-\frac{x}{a}}) - a \ldots (4)$. From (3) and (4) we get $a = (s^2 - y^2) \div 2y$.

From (2), we easily get $x=a\log_e[(y+\sqrt{y^2-a^2})/a]=a\log_e[(s+\sqrt{s^2+a^2})/a]=(s^2-y^2)/2y\log[(s+y)/(s-y)]\div\log_{10}e$.

$\therefore\ \log x=\log(s+y)+\log(s-y)+\log[\log(s+y)-\log(s-y)]-\log y-\log[2\log_{10}e]$.

$=\log(s+y)+\log(s-y)+\log[\log(s+y)-\log(s-y)]+\text{colog}\,y+0.0612$.

From this equation, since $x=40$m and $s=50$m, we find, by the Method of Double Position, the value of $y=26.53$m which is called the sag.

The tension at $A=wa=w\left(\frac{s^2-y^2}{2y}\right)=1559.78$ oz., and $\frac{FI}{EI}=\frac{a}{s}$ $=.6797$ the *batter*.

From the above equations we may obtain the four propositions as given in Wentworth and Hill's High School Arithmetic.

35. Proposed by B. F. FINKEL. Professor of Mathematics in Kidder Institute, Kidder, Missouri.

Between Sing-Sing and Tarry-Town, I met my worthy friend, John Brown,
And seven daughters, riding nags, and every one had seven bags:
In every bag were thirty cats, and every cat had forty rats.
Besides a brood of fifty kittens. All *but* the nags were wearing mittens!
Mittens, kittens—cats, rats—bags, nags—Browns,
How many were met between the towns?

[From *Mattoon's Common Arithmetic.*]

Solution by FRANK HORN, Columbia, Missouri.

II.
1. $8=$number of Browns met.
2. $8=8\times1=$number of nags.
3. $56=8\times7=$number of bags.
4. $1680=30\times56=$number of cats.
5. $67200=1680\times40=$number of rats.
6. $84000=1680\times50=$number of kittens.
7. $167888=$Browns+cats+rats+kittens.
8. $335776=167888\times2=$number of mittens worn provided that each person, cat, rat, and kitten wore one pair.
9. $636616=$Browns+nags+bags+cats+rats+mittens+kittens.

III. $\therefore$ The number of objects and persons met amounted to 636616.

NOTE.—The result given in Mattoon's Arithmetic is 2184192. What interpretation did Mr. Mattoon give to the problem?—EDITOR.

PROBLEMS.

42. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

If $m=2$ct. be the interest on $M=100$ct. for $p=10$ days, find the yearly rate per cent.

43. Proposed by B. F. BURLESON, Oneida Castle, New York.

A, in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar plums; B caught $\frac{3}{8}$ of it out of his hands, and C laid hold on $\frac{3}{10}$ more; D ran off with all A had left, except $\frac{1}{7}$ which E afterwards secured slyly for himself; then A and C jointly

they all went anew, for what it contained: of which, A got $\frac{1}{4}$, B $\frac{1}{3}$, and D $\frac{2}{7}$, and C and E equal shares of what was left of that stock. D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands; they, with difficulty, recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ apiece of the same. Upon this, they called a truce, and agreed that the $\frac{1}{3}$ of the whole, left by A at first, should be equally divided among them. How much of the prize, after this distribution, remained with each of the competitors?

set upon B, who, in the conflict, let fall $\frac{1}{2}$ he had, which were equally picked up by D and E, who lay perdu. B then kicked down C's hat, and to work

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

32. Proposed by LEV. WEINER, Professor of Modern Languages, Missouri State University, Columbia, Missouri.

Find a number consisting of 6 digits which when multiplied by the first 6 natural numbers gives the same digits in rotation.

I. Solution by LEONARD E DICKSON, M. A., Fellow in Mathematics, University of Chicago.

In a memoir on "Numbers with cyclic multiples" soon to be published, I have completely discussed general problems of which this is a very special case. One of my results is that there is only one number of more than one digit which when multiplied by as many different integers as the number contains digits each product has the same digits as the original number and in the same cyclic order. This number is 142857, which answers the problem.

$$\begin{aligned}\times 1 &= 142857\\ \times 2 &= 285714\\ \times 3 &= 428571\\ \times 4 &= 571428\\ \times 5 &= 714285\\ \times 6 &= 857142.\end{aligned}$$

Important to note is that the number $\times 7 = 999{,}999$. If in any of the above six multiples we add the number composed of the first three digits to that composed of the last three, the sum is 999.

II. Solution by the PROPOSER.

Let the digits be a, b, c, d, e, f, and let A, B, C, D, E be some one of the first 6 numbers but 1, respectively; then

$$A(10^5a+10^4b+\dots+f)=10^ra-10^{r-1}b+\dots+10^{n-r}f$$
$$B(10^5a+10^4b+\dots+f)=10^pa-10^{p-1}b+\dots+10^{n-p}f$$
$$\dots\dots\dots\dots\qquad\dots\dots\dots\dots$$
$$1(10^5a+10^4b+\dots+f)=10^5a+10^4b+\dots+f$$

$(1+A+B+\dots)(10^5a+10^4b+\dots+f)=(10^5+10^4+10^3+10^2+10^1+1)$ $(a+b+\dots+f)$.

$$\text{Now } 1+A+B+\dots=\frac{n(n+1)}{2}=21.$$

Hence, $10^5a+10^4b+\dots+e=\dfrac{111111}{21}(a+b+\dots+f)=5291(a+b+\dots+f)$.

By subtracting $a+b+\dots+e$ from both sides, we get $9(9999a+999b+99c+9d+e)=5290(a+b+\dots+f)$. Since the left side is divisible by 9, $a+b+c+\dots+f$ must be either 27 or 36, but 36 is readily seen to be impossible, since 5291×36 would give a number ending in 6, which when multiplied by six could not give the numbers in rotation; hence the only one to try is 27.

Now $5291\times27=142857$, and this number will be found to answer the purpose.

III. Solution by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

When a common fraction in its lowest terms is changed into a decimal fraction, and this decimal fraction is a pure circulator with a full period, that is, one which begins at the first decimal and contains a number of places by one smaller than the denominator of the common fraction, then the same period will occur, only commencing at a different figure of the period, for every fraction with the same denominator, but a different numerator. The fraction $\frac{1}{7}$ produces a pure circulator with a full period, consequently, according to the principle just mentioned, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ will produce the same period, only commencing at a different figure.

$\frac{1}{7}=.142857$, the next higher figure after 1 in the period is 2, $\therefore\ \frac{2}{7}=.285714$, the next higher figure is 4. $\therefore\ \frac{3}{7}=.428571$, etc. This answers the question proposed, the number being 142857.

IV. Solution by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania, and H. W. DRAUGHON, Olio, Mississippi.

It is clear that the first digit is 1, and this can only be the last digit when the multiplier is 3 in which case the last digit of the multiplicand must be 7. Now this 7 is to be multiplied by 2, 4, 5 and 6 and hence the other digits of the required number must be 4, 8, 5 and 2. Now 8 being the largest digit must be the first digit in the product when 6 is the multiplier and hence dividing 8 by 6, we get 4 as the second digit of the required number. Now assume 8 or 5 as the third digit and multiply by 5; the digits can not be brought in the required order; hence the third digit is 2 and the number is 142857.

33. Proposed by C. E. WHITE, Trafalgar, Lndiana.

Show that every algebraic equation of the nth degree, n being greater than two, which is complete in its terms may be transformed into an infinite number of equations which want their second term.

Solution by the PROPOSER.

It is shown in treatises on Higher Algebra that an equation may be changed into another equation of the same degree, but which wants its second term. Now let $x^n+mx^{n-2}+nx^{n-3}+\ldots\ldots+px^2+qx+r=0$ represent this first derived equation, and let

$(x^{n-1}+ax^{n-2}+bx^{n-3}+cx^{n-4}+\ldots\ldots+fx^2+gx+h)(x-a)=0$ represent it as factored. Equating coefficients, we have

$$m=-a^2+b\ldots.(1)$$
$$n=-ab+c\ldots.(2)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$p=-af+g\ldots.(n-3)$$
$$q=-ag+h\ldots.(n-2)$$
$$r=-ah\ldots.(n-1).$$

Now it is easily seen that we can derive without difficulty an equation in h by eliminating a from $(n-1)$ and g from $(n-2)$ and f from $(n-3)$, etc. Moreover, since $h=-\frac{r}{a}$, the derived equation in h will contain its second term.

Now, by the method by which we derived the first equation, we may derive from the equation in h another equation of the same degree wanting its second term. From this second derived equation we may derive a third, etc. Hence by continuing the process it is possible to derive an infinite number of such equations. Moreover, the first derived equation may be considered as derived from preceding equations; hence we may find an infinite number of preceding equations of the same form. By every transformation we change the value of the constant term; hence, it is possible that it may take the value zero in one of the derived equations, thus enabling us to find, at least, one root of the equation. Consider the cubic equation, and let x^3+qx+r represent its first derived equation, then, $x^3-\frac{1}{3}q^2x-\frac{2}{27}q^3-\frac{1}{4}r^2=0$ will represent its second derived equation Now, if q be negative and $\frac{2}{27}q^3=\frac{1}{4}r^2$, as in the particular case $x^3-6x+4=0$, the second derived equation reduces to the form $x^3-\frac{1}{3}q^2x=0$. Whence, $x=0$, $\pm\frac{1}{\sqrt{3}}q$, from which we easily find the roots of the first derived to be $x=-\frac{3r}{q}$, $\frac{3(1+\sqrt{3})r}{2q}$, and $\frac{3(1-\sqrt{3})r}{2q}$

It should be observed that all cubic equations that can be so resolved belong to the irreducible case.

Also solved by *H. C. Whitaker*.

PROBLEMS.

44. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

Find the general term in the series 1, 3, 10, 35, 126, 462, 1716, 6435, 24310, ...,which plays a remarkable part in some recent theorems in my theory of Regular Polygons.

45. Proposed by WILLIAM HOOVER, A. M., Ph. D., Ohio State University, Athens, Ohio.

Find x from $\cos^{-1}\dfrac{1-x^2}{1+x^2}+\tan^{-1}\dfrac{2x}{1-x^2}=\dfrac{4\pi}{3}$.

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

32. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio State University, Athens, Ohio.

If a conic be incribed in a triangle and its focus move along a given straight line, the locus of the other focus is a conic circumscribing the triangle.

I. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Using trilinear co-ordinates the equation to the inscribed ellipse is of the form $\sqrt{(l\alpha)}+\sqrt{(m\beta)}+\sqrt{(n\gamma)}=0$.

Let α' β' γ', be the co-ordinates of the one focus, then

$\dfrac{\alpha}{\alpha'}=\dfrac{\beta}{\beta'}$, $\dfrac{\beta}{\beta'}=\dfrac{\gamma}{\gamma'}=\dfrac{\alpha}{\alpha'}$ are the equations to the lines joining it to the vertices of the triangle. The lines to the other focus make equal angles with the sides of the triangles, hence, their equations are $\alpha'\alpha=\beta'\beta$, $\beta'\beta=\gamma'\gamma$, $\gamma'\gamma=\alpha'\alpha$. $\therefore$ the co-ordinates of the other focus may be taken

$\dfrac{1}{\alpha'}$, $\dfrac{1}{\beta'}$, $\dfrac{1}{\gamma'}$; from this relation, if we are given the equation of any locus described by one focus, we can at once write down the equation of the locus described by the other focus.

$\therefore$ If the first focus describes the straight line $l\alpha+m\beta+n\gamma=0$, the second will describe the locus whose equation is

$$\frac{l}{\alpha}+\frac{m}{\beta}+\frac{n}{\gamma}=0, \text{ a conic circumscribing the triangle.}$$

II. **Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio State University, Athens, Ohio.**

In trilinear co ordinates, let the foci be $(\alpha',\beta',\gamma'),(\alpha'',\beta'',\gamma'')$. Then since the product of the perpendiculars from the foci upon tangents to a conic is constant, we should have $\alpha'\alpha''=\beta'\beta''=\gamma'\gamma''=k\ldots.(1)$.

If $l\alpha+m\beta+n\gamma=0\ldots.(2)$ be the locus of (α',β',γ'), it is plain from (1) that $\frac{l}{\alpha''}+\frac{m}{\beta''}+\frac{n}{\gamma''}=0\ldots.(3)$, or $l\beta\gamma+m\alpha\gamma+n\alpha\beta=0\ \ldots(4)$, by dropping accents, which is a circumscribing conic.

33. **Proposed by Professor B. F. SINE, Shenandoah Normal College, Reliance, Virginia.**

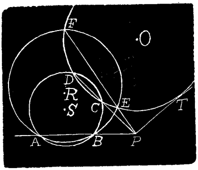
If a given circle is cut by another circle passing through two fixed points the co mmon chord passes through a fixed point.

I. **Solution by GEORGE R. DEAN, C. E., B. Sc., High School. Kansas City, Missouri.**

The straight line containing the two given points is the radical axis of every pair of circles to which the points are common. Let the radical axis of the given circle and one of these circles intersect the given radical axis at some point O; then the radical axis of the given circle and any other circle containing the given points must pass through O, for the radical axis of three circles meet in a point.

II. **Solution by J. C. GREGG, Superintendent of Schools, Brazil, Indiania; and P. S. BERG, Apple Creek, Ohio.**

Let A and B be two fixed points and O the center of a fixed circle. Let R be the center of any circle through A and B and cutting circle O in D and C.



To show that the chord DC passes through a fixed point. Produce AB and DC to meet in P; then P is the required point. Draw the tangent PT. Then we have $PA.PB=PD.PC=\overline{PT}^2\ldots.(1)$.

Draw any other circle (center S) through A and B and cutting circle O, in two points one of which is E. Draw PE and produce it till it cuts circle S in some point X and O in F. Now from the secants PA and PX we have $PE.PX=PA.PB=\overline{PT}^2$ from (1) and from secant PF and tangent PT we have $PE.PF=\overline{PT}^2$. $\therefore$ $PE.PX=PE.PF$ and hence $PX=PF$ and the points X and F coincide and are the intersection of circles S and O

and the chord FE passes through P; and so for any circle. Q. E. D.

This problem was also solved by *G. B. M. Zerr, John B. Faught, J. F. W. Scheffer*, and *O. W. Anthony*.

PROBLEMS.

37. Proposed by B. F. BURLESON, Oneida Castle, New York.

Inscribe in a semicircle a rectangle having a given area; a rectangle having the maximum area.

38. Proposed by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

Give a *strictly geometric* proof of my fundamental theorem on the Inscription of Regular Polygons, viz: Suppose a circle of unit radius divided at the points $A, A_1, A_2, A_3, \dots A_p, \dots$ into $2p+1$ equal parts and the diameter AO drawn. Then, if the chords $OA_1, OA_2, \dots OA_p$ be drawn, we have $OA_1 - OA_2 + OA_3 - OA_4 + OA_5 - \dots \pm OA_p = 1$.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS

25. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

A leaf of the curve: "The Devil on Two Sticks", equation $y^4 - x^4 + 100a^2x^2 - 96a^2y^2 = 0$, revolves around the axis of x. Deduce the expression for the volume generated.

I. Solution by the PROPOSER.

From the equation of the given curve, we deduce $y^2 = 48a^2 \pm \sqrt{(2304a^4 - 100a^2x^2 + x^4)}\dots(1)$; that is, $(PD)^2 = 48a^2 + \sqrt{(2304a^4 - 100a^2x^2 + x^4)}$, and, therefore, $(P'D)^2 = 48a^2 - \sqrt{(2304a^4 - 100a^2x^2 + x^4)}$. Hence the expression for the volume generated after the curve has made a complete revolution around the axis of x, becomes

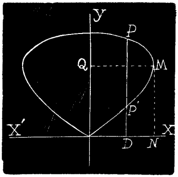
$$V = 2\pi\left[\int_0^{6a}[48a^2 + \sqrt{(2304a^4 - 100a^2x^2 + x^4)}]dx - \int_0^{6a}[48a^2 - \sqrt{(2304a^4 - 100a^2x^2 + x^4)}]dx\right]\dots(2).$$

Condensing (2), then factoring, etc.,

$$V=4\pi\int_0^{6a}\sqrt{(2304a^4-100a^2x^2+x^4)}dx=192\pi a^2$$

$$\int_0^{6a}\sqrt{\left[\left(1-\frac{x^2}{36a^2}\right)\left(1-\frac{x^2}{64a^2}\right)\right]}dx\ldots.(3).$$

I. Let $x^2/36a^2=w^2$; then will $x^2/64a^2=\frac{9}{16}w^2,=c^2w^2$, and $dx=6adw$. Making these substitutions in (3), we have



$$V=1152\pi a^3\int_0^1\sqrt{[(1-w^2)(1-c^2w^2)]}dw,$$

$$=1152\pi a^3\int_0^1\frac{1-(1+c^2)w^2+c^2w^4}{\sqrt{[(1-w^2)(1-c^2w^2)]}}dw$$

$$=384\pi a^3\int_0^1\left[\frac{1-2(1+c^2)w^2+3c^2w^4}{\sqrt{[(1-w^2)(1-c^2w^2)]}}\right.$$

$$\left.+\frac{2-(1+c^2)w^2}{\sqrt{[(1-w^2)(1-c^2w^2)]}}\right]dw$$

$$=384\pi a^3\left[\int_0^1\frac{1-2(1+c^2)w^2+3c^2w^4}{\sqrt{[(1-w^2)(1-c^2w^2)]}}dw\right.$$

$$\left.+\left(\frac{1+c^2}{c^2}\right)\int_0^1\frac{\sqrt{(1-c^2w^2)}}{\sqrt{(1-w^2)}}dw-\left(\frac{1-c^2}{c^2}\right)\int_0^1\frac{dw}{\sqrt{[(1-w^2)(1-c^2w^2)]}}\right]$$

$$=384\pi a^3\left\{\left[w\sqrt{(1-w^2)(1-c^2w^2)}\right]_0^1+\left(\frac{1+c^2}{c^2}\right)\left[\mathbf{E}(c,w)_0^1\right.\right.$$

$$\left.\left.-\left(\frac{1-c^2}{1+c^2}\right)\mathbf{F}(c,w)_0^1\right]\right\}\ldots.(4).$$

II. By making $x^2/36a^2=\sin^2\psi$, $x^2/64a^2=\frac{9}{16}\sin^2\psi,=c^2\sin^2\psi$, and $dx=6a\cos\psi d\psi$, we easily deduce from (3) the following expression:

$$V=1152\pi a^3\int_0^{\frac{1}{2}\pi}\cos^2\psi\sqrt{(1-c^2\sin^2\psi)}d\psi=384\pi a^3\left(\frac{1+c^2}{c^2}\right)\left[\mathbf{E}(c,\tfrac{1}{2}\pi)\right.$$

$\left.-\left(\frac{1-c^2}{1+c^2}\right)\mathbf{F}(c,\tfrac{1}{2}\pi)\right]\ldots.(5)$, which is identical with the right-hand member of (4). As a *working-result*, the right-hand member of (5) is preferable to the right hand member of (4). Expanding the Legendrian elliptic-integrals in the right-hand member of (5), uniting corresponding terms, etc., we have $V=288\pi^2a^3[1-\frac{1}{8}c^2-\frac{1}{64}c^4-\text{etc.}]=\frac{136359}{512}\pi^2a^3\ldots.(6)$. Putting $a=1$ and remembering that $\pi^2=9.8696+$, we obtain $V=2628.533+$.

II. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

The polar equation to the curve is,

$$r^2=\frac{4a^2(24\sin^2\theta-25\cos^2\theta)}{\sin^4\theta-\cos^4\theta}=\frac{4a^2(24\sin^2\theta-25\cos^2\theta)}{\sin^2\theta-\cos^2\theta}=\frac{2a^2(1+49\cos 2\theta)}{\cos 2\theta}.$$

Both leaves are equal and each is symmetrical with reference to the y axis. Also, the area of the upper leaf is comprised between the limits $\theta=\frac{\pi}{4}$ and $\theta=\frac{3\pi}{4}$. Let A=area of this leaf, $\bar{y}$=ordinate of its centroid. Then volume required is $V=2\pi\bar{y}A$.

$$A=\tfrac{1}{2}\int r^2\,d\theta=a^2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{1+49\cos 2\theta}{\cos 2\theta}\,d\theta=\frac{49\pi a^2}{2}.$$

$$\bar{y}=\tfrac{2}{3}\frac{\int r^3\sin\theta d\theta}{\int r^2 d\theta}=\frac{2a\sqrt{2}}{3}\cdot\frac{\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left\{\frac{1+49\cos 2\theta}{\cos 2\theta}\right\}^{\frac{3}{2}}\sin\theta d\theta}{\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{1+49\cos 2\theta}{\cos 2\theta}d\theta}.$$

$$\bar{y}=\frac{4a\sqrt{2}}{147\pi}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left\{\frac{1+49\cos 2\theta}{\cos 2\theta}\right\}^{\frac{3}{2}}\sin\theta d\theta.$$

$\therefore\quad V=\dfrac{4\pi a^3\sqrt{2}}{3}\displaystyle\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left\{\frac{1+49\cos 2\theta}{\cos 2\theta}\right\}^{\frac{3}{2}}\sin\theta d\theta.$ Let $\sqrt{2}\cos\theta=\cos\phi$;

$$\therefore\quad V=\frac{4\pi a^3}{3}\int_0^{\pi}(48-49\cos^2\phi)^{\frac{3}{2}}\operatorname{cosec}^2\phi d\phi=196\pi a^3\int_0^{\pi}(48-49\cos^2\phi)^{\frac{1}{2}}\cos^2\phi d\phi$$

$=784\sqrt{3}\pi a^3\displaystyle\int_0^{\pi}(1-c\cos^2\phi)^{\frac{1}{2}}\cos^2\phi d\phi$, where $c=\dfrac{49}{48}$,

$$=784\sqrt{3}\pi a^3\int_0^{\pi}(1-\tfrac{1}{2}c\cos^2\phi-\tfrac{1}{8}c^2\cos^4\phi-\tfrac{1}{16}c^3\cos^6\phi-\tfrac{5}{128}c^4\cos^8\phi-\text{, etc.})\cos^2\phi d\phi$$

$$=392\sqrt{3}\pi^2a^3\left\{1-\frac{3}{8}\left(\frac{49}{48}\right)-\frac{5}{64}\left(\frac{49}{48}\right)^2-\frac{35}{1024}\left(\frac{49}{48}\right)^3-\frac{315}{16384}\left(\frac{49}{48}\right)^4-\text{, etc.}\right\}$$

$=392\sqrt{3}\pi^2a^3\cdot\left\{1-3b-5b^2-\dfrac{35}{2}b^3-\dfrac{315}{4}b^4-\ldots\ldots\right\}$, where $b=\dfrac{49}{384}$.

The fifth term of this serie $=\frac{1}{50}$ nearly. Or thus: but since $c=e^2$ is not less than unity, let $x=\cos\phi$, then

$$\int_0^{\pi}(1-\cos^2\phi)^{\frac{1}{2}}\cos^2\phi d\phi=2\int_0^{1}\left\{\frac{e^2x^2-1}{x^2-1}\right\}^{\frac{1}{2}}x^2dx=2I.$$

Let $S=\sqrt{(e^2x^2-1)(x^2-1)}$, then $d\,(Sx)=\left(S+\dfrac{2e^2x^4-x^2(e^2+1)}{S}\right)dx$

$$=\frac{1+3e^2x^4-2x^2(e^2+1)}{S}dx=\frac{1-(2e^2-1)x^2}{S}dx+3dI=\frac{1-\frac{2e^2-1}{e^2}}{S}dx$$

$$-\frac{2e^2-1}{e^2}\,d[H(e,x)]+3dI=-\frac{e^2-1}{e^2}\,d[H'(e,x)]-\frac{2e^2-1}{e^2}d[H(e,x)]+3dI.$$

$$\therefore\left[x\sqrt{(e^2x^2-1)(x^2-1)}+\frac{e^2-1}{e^2}H'(e,x)+\frac{2e^2-1}{e^2}H(e,x)\right]_0^1=3I.$$

$$2I=\left[\frac{2}{3}\cdot\frac{e^2-1}{e^2}H'(e,x)+\frac{2}{3}\cdot\frac{2e^2-1}{e^2}H(e,x)\right]_0^1.$$

$$\therefore\ V=\frac{1568\sqrt{3}\pi a^3}{3e^2}\left[\frac{2e^2-1}{e^2}H(e,x)+\frac{e^2-1}{e^2}H'(e,x)\right]_0^1,\ \text{but } e^2=\frac{49}{48}.$$

$\therefore\ V=\frac{512\sqrt{3}\pi a^3}{49}\left[50H\left(\frac{7}{4\sqrt{3}}\right)+H'\left(\frac{7}{4\sqrt{3}}\right)\right]$, where H and H' denote the hyperbolic functions corresponding to the elliptic functions E and F.

III. Remarks by Professor J. F. W. SCHEFFER. A. M., Hagerstown, Maryland.

This curve, called in French "la courbe du diable," is of the middle point of a chord to the equilateral hyperbola $x^2-y^2=2a^2$, the chord being of constant length and equal to seven times the transverse axis $2a\sqrt{2}$. Its equation is found thus: Let (x_1, y_1) and (x_2, y_2) be the extremities of a chord, and (x, y) any point of the curve, then we have the equations: $x^2-y^2=2a^2 \ldots (1)$. $x^2_2-y^2_2=2a^2 \ldots (2)$, $2x=x_1+x_2 \ldots (3)$, $2y=y_1+y_2 \ldots (4)$, and $(x_1-x_2)^2+(y_1-y_2)^2=392a^2 \ldots (5)$. Subtracting (2) from (1), and considering (3) and (4), we have $(x_1-x_2)x=(y_1-y_2)y$, whence $y_1-y_2=(x_1-x_2)\frac{x}{y}$. Substituting this in (5), we get $x_1-x_2=\frac{14\sqrt{2}.ay}{(x^2+y^2)^{\frac{1}{2}}}$, $y_1-y_2=\frac{14\sqrt{2}.ax}{(x^2-y^2)^{\frac{1}{2}}}$, and combining these with (2) and (3), we get $x_1=x+\frac{7\sqrt{2}.ay}{(x^2+y^2)^{\frac{1}{2}}}$, $y_1=y+\frac{7\sqrt{2}.ax}{(x^2+y^2)^{\frac{1}{2}}}$. Substituting in (1) and simplifying we finally have $y^4-x^4-96a^2y^2+100a^2x^2=0$.

Query: Can any one furnish a reason for the peculiar name of the "devil's curve," or the name which Prof. Matz employes?

Also solved by *Prof. C. W. M. Black*.

PROBLEMS.

34. Proposed by GEORGE LILLEY, Ph. D., LL. D., Park School, Portland, Oregon.

A hare is at O, and a hound at E, 40 rods east of O. They start at the same instant each running with uniform velocity. The hare runs north. The hound runs directly towards the hare and overtakes it at N, 320 rods from O. How far did the hound run?

35. Proposed by H. C. WHITAKER, B. S., C. E., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Water is running into a vessel in the shape of a frustum of a cone (radii up-

per and lower bases 15 inches and 10 inches, respectively, and altitude 20 inches) at the rate of 10 cubic inches per second. When the depth is 8 inches at what rate is it increasing?

MECHANICS.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

14. Proposed by ALFRED HUME, C. E., Sc. D., Professor of Mathematics, University of Mississippi, University P. O., Miss.

"The center of a sphere of radius C moves in a circle of radius A and generates thereby a solid ring, as an anchor-ring: prove that the moment of inertia of this ring about an axis passing through the center of the direct circle and perpendicular to its plane is $\frac{1}{4}\pi^2\delta ac^2(4a^2+3c^2)$."

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

If the moment axis be the axis of z, the origin being the center of the ring, and the axis of x and y any two diameters at right angles the required moment could be obtained from $\int\int(x^2+y^2)zdxdy$, having the equation to the surface of the ring.

But the following method quoted by Williamson in the Int. Cal., New York Edition, 1884, art. 212 from Townsend is so concise I prefer to give it.

Let y, Y be the distances of any point in the meridian section of the sphere from that diameter of the section parallel to the moment axis, and to the moment axis. Then if dA be the element of area of the generating section, the mass of the elementary ring generated by dA is $2\pi\mu YdA$, and the moment of inertia of this ring is $2\pi\mu Y^3dA$.

$$\therefore \text{ the required } M.I.=2\pi\mu\int Y^3dA=2\pi\mu\int(a+y)^3dA$$

$$=2\pi\mu\int(a^3+3a^2y+3ay^2+y^3)dA\ldots.(1).$$

But from theory, $\int ydA=0$, $\int y^3dA=0$, and if k be the radius of gyration of the generating section, $\int y^2dA=Ak^2$; then (1) becomes

$$M.I.=2\pi\mu aA(a^2+3k^2)=2\pi^2\mu ac^2(a^2+\tfrac{3}{4}c^2)$$

$=\frac{\pi^2\mu ac^2}{2}(4a^2+3c^2)\ldots(2)$, in which μ is used instead of δ, and the result is twice as great as given in the statement of the problem.

The advantage of this method lies in the fact that it is general for A and k^2, which are therefore the only quantities to be worked out before setting down the special result.

II. **Solution by F. P. MATZ, M Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

According to well-known principles $V=\pi a^2\times 2\pi c-2\pi^2ac^2$, $M=V\delta=2\pi^2\delta ac^2$, and the Radius of Gyration$=X=\surd(\frac{1}{2}a^2+\frac{3}{8}c^2)$. Hence the required Moment of Inertia, *Nystrom's Mechanics*, becomes $K=MX^2$ $=\frac{1}{4}\pi^2\delta ac^2(4a^2+3c^2)$.

III. **Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics in the University of Mississippi.**

If the center of the generating circle be taken as the origin and a perpendicular from this point to the axis of revolution as the axis of x, the equation of the moving circle is $x^2+y^2=c^2$.

Divide the ring formed into layers of infinitesimal thickness, dy, by planes parallel to the plane of the director circle.

The moment of inertia of any layer whose external radius is $a+x$ and internal $a-x$ is $\left[\frac{\pi}{2}\rho(a+x)^4-\frac{\pi}{2}\rho(a-x)^4\right]dy$, ρ being the density.

Therefore the moment of inertia of the entire ring is

$4\pi\rho a\int_{-c}^{c}(a^2+c^2-y^2)(c^2-y^2)^{\frac{1}{2}}dy$, substituting c^2-y^2 for x^2.

Performing the integration the result is $\frac{\pi^2\rho ac^2}{2}(4a^2+3c^2)$ which is double that given by Price.

This problem was also solved by *W. Wiggins, G. B. M. Zerr, and P. H. Philbrick*. Their solutions will be published next month.

PROBLEMS.

20. **Proposed by CHAS. E. MYERS, Canton, Ohio.**

A flexible cord of given length is suspended from two points whose co-ordinates are (x, y) and (x', y'). What must be the condition of the cord in order that it may hang in the arc of a circle?

21. **Proposed by J. A. CALDERHEAD, Superintendent of Schools, Limaville, Ohio.**

Show that, in the wheel and axle, when a force P, acting at the circumference of the wheel, supports a weight Q upon the axle,

$$P.(R \mp \rho \sin\varepsilon) = Q(r \pm \rho \sin\varepsilon) \pm W\rho \sin\varepsilon,$$

where R, r, and ρ are the radii of the wheel, the axle, and their common axis respectively, and ε is the limiting angle of resistance.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

16. Proposed by H. W. DRAUGHON, Olio, Mississippi.

Find three numbers such that the cube of any one plus the sum of the squares of the other two, will be a square.

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Let ax, bxy, and cxz represent the required numbers; then we have $a^3x^3+(b^2y^2+c^2z^2)x^2=\square\ldots.(A)$, $b^3x^3y^3+(a^2+c^2z^2)x^2=\square\ \ldots(B)$, and $c^3x^3z^3+(a^2+b^2y^2)x^2=\square\ldots.(C)$. Omitting in (A), (B), and (C), the factor x^2, and putting $x=2bcyz/a^3$, we have (A) as a perfect square.

Substitute this value of x in the first term of (B); then, obviously, the condition that (B) will be a perfect square, is $2b^4cy^4z/a^3=2acz$.

$\therefore y=a/b\ldots.(1)$. After performing a similar operation in (C), we obtain $2bc^4yz^4/a^3=2aby$. $\therefore\ z=a/c\ldots.(2)$.

Consequently $x=2bcyz/a^3=2/a\ldots.(3)$; and the required numbers are $ax=2$, $bxy=2$, and $cxz=2$.

[NOTE.—Can any of our contributors find three *unequal* numbers answering the conditions of this problem? The proposer and several contributors have reported that they had as yet failed to solve it. The problem seems difficult of solution, or at least the EDITOR does not now see any way clear to a solution of it.]

17. Proposed by ARTEMAS MARTIN, LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Is it possible to find two positive whole numbers such that each of them and also their sum and difference, when diminished by unity shall all be squares?

Solution by the PROPOSER.

Let x^2+1 and y^2+1 denote the numbers required; then their sum $=x^2+y^2+2$, their difference $=x^2-y^2$, and we have

$$x^2+y^2+1=\square=v^2\ \ldots.(1),$$
$$x^2-y^2-1=\square=w^2\ \ldots.(2).$$

From the first of these equations, $y^2=v^2-x^2-1\ldots.(3)$. Adding (1) and (2) we get $2x^2=v^2+w^2\ldots.(4)$. Let $v=t+u$, $w=t-u$ and (4) becomes $x^2=t^2+u^2\ldots.(5)$, which is satisfied by

$t=p^2-q^2$, $u=2pq$, $x=p^2+q^2$, and then $v=p^2+2pq-q^2$.

Substituting these values of x and v in (3) we get

$$y^2=4pq(p^2-q^2)-1\ldots.(6).$$

As the right hand member of (6) is of the form $4m-1$ it can not be an integral square, and therefore the problem is impossible.

II. **Solution by C. A. ROBERTS, Long Bottom, Ohio.**

Let (x^2+1) and (y^2+1) be the numbers, which when diminished by 1 give x^2 and y^2.

Let $(x^2+1)-(y^2+1)-1=a^2=x^2-y^2-1$, which call (I);

Let $(x^2+1)+(y^2+1)-1=b^2=x^2+y^2+1$, which call (II).

The square root of an even square, is even; the square root of an odd square, is odd. Take (I), $a^2=x^2-y^2-1$, and transposing, $a^2+y^2+1=x^2$ or three squares whose sum is a square. The square of any even number is divisible by 4, without a remainder, and is therefore said to be of the form of $(4n)$. If the square of any odd number be divided by 4, there will be a remainder of 1, and such squares are said to be of the form of $(4n+1)$. Any number not of the form of $(4n)$ or $(4n+1)$ is not a square. Let us determine in the equation $a^2+y^2+1=x^2$ whether a and y, are both odd, both even, or one of them odd and the other even.

1 is odd and of the form of $(4n+1)$; if a is even and y odd, or if y is even and a odd, we have for the form of the sum $(4n)+(4n+1)+(4n+1)$ $=(12n+2)$, which is of the form of $(4n_1+2)$ and which can not be a square. If a and y are both odd, we have for the form of the sum, $(4n+1)+(4n+1)$ $+(4n+1)=12n+3$, which is of the form of $(4n_1+3)$, and which can not be a square. If a and y are both even, we have for the form of the sum, $(4n)+(4n)+(4n+1)=(12n+1)$, which is of the form of $(4n_1+1)$, and may be a square, and if $(4n_1+1)$ is a square, as x^2, it is an odd square, and x is odd. Therefore in order that the equation $a^2+y^2+1=x^2$ shall be true in integers, a and y *must* be even numbers, and x *must* be an odd number.

Take (II) $x^2+y^2+1=b^2$; applying to (II) the reasoning in (I) x and y *must* be even, and b odd, or x must be both odd, (as in I) and even, (as in II). As this is impossible with the same value of x, there are no such numbers as called for in the problem.

[NOTES.—CHAS. DE MEDICI of 60 West 22nd St., New York, overlooking the punctuation of the problem as published, gives 64 and 81 as the numbers, and adds an interesting exhibit showing that the curio of these values are not by any means limited to what the question, as he read it, asked for.

M. A. Gruber, P. H. Philbrick, and G. B. M. Zerr, should have been credited for solving problem 15, December Number. Their solutions were

selected for publication, but owing to the fact the December Number had to be cut short in order to get it out without further delay their solutions were omitted.—EDITOR.]

PROBLEMS.

25. **Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Find, if possible, integral values of each of the seven linear measurements of a rectangular parallelopiped; i. e. length, breadth, height, the diagonals of each of the three different rectangular sides, and the diagonal from an upper corner to the opposite lower corner; or, find integral values, if possible, of a, b, c, d, e, f, and g, as shown in the equations,—$a^2+b^2=c^2$, $a^2+d^2=e^2$, $a^2+f^2=g^2$, $b^2+d^2=f^2$, $b^2+e^2=g^2$, $c^2+d^2=g^2$, $c^2+e^2=f^2$. If not possible, how many of them can have integral values? and which?

26. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Find (1) a *square fraction* the arithmetical difference of whose terms is a *cube*; and (2) find a *cubic fraction* the arithmetical sum of whose terms is a *square*.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

12. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

A large plane area is ruled by two sets of parallel equidistant straight lines, the one set perpendicular to the other. The distance between any two lines of the first set is a; the distance between any two lines of the second set is b. If a regular polygon of $2n$ sides be thrown at random upon this area, find the chance that it will fall across a line, the diameter of the circum-circle of the polygon being less than a or b.

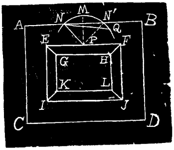
II. **Solution by H. W. DRAUGHON, Clinton, Louisiana.**

In the rectangle $ABCD$ let $AB=a$ and $AC=b$. Let c=apothem of polygon, and r=radius of its circum-circle.

Let the sides of the rectangle $EFIJ$, be parallel to, and distant c, from the corresponding sides of $ABCD$, and let the sides of the similarly

placed rectangle $GHKL$, be r distant from the corresponding sides of AB. Draw EG, FH, LJ, and IK.

1. If the center of the polygon falls without the rectangle $EFIJ$, the polygon will cross a line in every possible position. ∴ the number of favorable positions for this case is, $n_1 = 2\pi r \times$ area of surface on which center falls $= 2\pi r[ab-(a-2c)(b-2c)]$. 2. Let us supposethat the center falls on the point P within the trapezoid $EFGH$. From P as a center, radius r, draw an arc cutting AB in the points N and N'. Draw one side of polygon NQ, also draw PM, perpendicular to AB. Put $AM=x$ and $PM=y$. The number of favorable positions for the point P, is obviously, $2n \times \text{arc}NN' = 4nr\cos^{-1}\left(\frac{y}{r}\right)$.



When x varies from $a-r$ to r, y can have any value from r to c. When x varies from r to c, y can have any value from x to c. The integration between the remaining limits for x and y, will obviously give the same result as that between last mentioned limits.

∴ in this case, the total number of favorable positions is,

$$n_2 = 4nr\left[2\int_c^r\int_c^x \cos^{-1}\left(\frac{y}{r}\right)dxdy + \int_r^{a-r}\int_c^r \cos^{-1}\left(\frac{y}{r}\right)dxdy\right]$$

$$= 4nr\left[2\int_c^r \left(x\cos^{-1}\left(\frac{x}{r}\right)dx - \sqrt{(r^2-x^2)}dx - c\cos^{-1}\left(\frac{c}{r}\right)dx\right.\right.$$

$$\left.\left. + \sqrt{(r^2-c^2)}\,dx + \int_r^{a-r}\left(-c\cos^{-1}\left(\frac{c}{r}\right)dx + \sqrt{(r^2-c^2)}\,dx\right)\right]$$

$$= 4nr\left\{2\left[\left(\frac{x^2}{2}-\frac{r^2}{4}\right)\cos^{-1}\left(\frac{x}{r}\right) - r^2\cos^{-1}\left(\frac{x}{r}\right) + \frac{x}{2}\sqrt{(r^2-x^2)}\right.\right.$$

$$\left.\left. + \frac{r^2}{2}\cos^{-1}\left(\frac{x}{r}\right)\right]_c^r + \left[-cx\cos^{-1}\left(\frac{c}{r}\right) + x\sqrt{(r^2-c^2)}\right]_r^{a-r}$$

$$= 4nr\left[\left(\tfrac{3}{2}r^2 + 2cr - ac - c^2\right)\cos^{-1}\left(\frac{c}{r}\right) + (a-2r-c)\sqrt{(r^2-c^2)}\right].$$

If the center falls within rectangle $GHKL$, the polygon can not cross a line.

3. The total number of favorable positions, when the center falls within the trapezoid $GKEI$ is found by changing a to b in the value of n_2. ∴ we have for this case, number of favorable positions,

$$n_3 = 4n\left[\left(\tfrac{3}{2}r^2 + 2cr - bc - c^2\right)\cos^{-1}\left(\frac{c}{r}\right) + (b-2r-c)\sqrt{(r^2-c^2)}\right].$$

4. If the center falls within the trapezoids $KLIJ$ or $HLFJ$, the number of favorable positions is, respectively, n_2 and n_3. ∴ when the center falls within rectangle $ABCD$, the number of favorable positions is,

$n_4 = n_1 + 2n_2 + 2n_3 = 2\pi r[ab-(a-2r)(b-2r)]$

$$+8nr[(\tfrac{3}{2}r^2+2cr-ac-c^2)\cos^{-1}\left(\frac{c}{r}\right)+(a-2r-c)\sqrt{(r^2-c^2)}]$$

$$+8nr[(\tfrac{3}{2}r^2+2cr-bc-c^2)\cos^{-1}\left(\frac{c}{r}\right)+(b-2r-c)\sqrt{(r^2-c^2)}]$$

$$=2\pi r[ab-(a-2r)(b-2r)]+8nr[(3r^2+4cr-(a+b)c-2c^2)$$

$$\cos^{-1}\left(\frac{c}{r}\right)+(a+b-4r-2c)\sqrt{(r^2-c^2)}].$$

Let S be the number of rectangles, ab, then the probability required is, $P=Sn_4\div 2\pi rSab=n_4\div 2\pi rab=\{\pi[ab-(a-2r)(b-2r)+4n[(3r^2+4cr$

$$-(a+b)c-2c^2)\cos^{-1}\left(\frac{c}{r}\right)+(a+b-4r-2c)\sqrt{(r^2-c^2)}]\}\div\pi ab.$$

PROBLEMS.

24. Proposed by F. P. MATZ, M. Sc. Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

The average area of the triangle formed by three perpendiculars drawn from the sides of the triangle (a, b, c), is $\mathbf{A}=(a^4+b^4+c^4)/48\triangle$.

25. Proposed by G. B. M. ZERR, Principal of High School, Staunton, Virginia.

The probability that the distance of two points taken at random in a given convex area A shall exceed a given limit (a) is

$$\triangle=\frac{1}{3A^2}\int\int(C^3-3a^2C+2a^3)dpd\theta,$$

where C is a chord of the area, whose co-ordinates are p, θ; the integration extending to all values of p, θ, which give a chord $C>a$. What is $\triangle$ when the area is a circle? If in the circle $a=r=$radius $\triangle=\frac{3\sqrt{3}}{4\pi}$.

INFORMATION.

PROFESSOR ARTHUR CAYLEY DEAD.

The Distinguished English Mathematician Passes Away at Cambridge.

LONDON, Jan. 31.—Prof. Arthur Cayley, of the University of Cambridge, died to-day, in the seventy-fourth year of his age. He had been for thirty-two years Sadlerian professor of pure mathematics at Cambridge, and

was regarded by educators everywhere as one of the three greatest of co-temporary mathematicians, the others being Professor Sylvester, of Oxford, and Professor Klein, of Goettingen.

Before becoming a professor at Cambridge he had been for fourteen years a conveyancer at Lincoln's Inn, London. He had been educated at King's College, London, and at Trinity College, Cambridge, where he graduated as Senior wrangler in 1842, and was shortly after elected a fellow.

The scientific writings of Professor Cayley relate to every branch of pure mathematics beside dynamics and astronomy.

The honors which the dead mathematician received from his own university and other universities, from English and foreign societies and from foreign governments can be numbered by the score and include honorary degrees, medals, fellowships and other honorary positions. The French government in 1890 made him an officer of the Legion of Honor, and he was also a correspondent of the French Institute and the winner of the Copley and Royal medals from the Royal Society, of which he is a fellow.

The higher education of women was one of the many practical problems in which he was interested, and for a number of years he had been chairman of the association to which belongs Newnham College, at Cambridge.

AT THE JOHNS HOPKINS UNIVERSITY.

By request of Professor J. J. Sylvester, Professor Arthur Cayley, the Sadlerian professor of pure mathematics of Cambridge, England, was associated in the mathematical work of the Johns Hopkins University, from January to June, 1882. Professor Sylvester, now the Savilian professor of Geometry in the University of Oxford, England, was then at the head of the Department of Mathematics in the Johns Hopkins University; and the presence in Baltimore, at the same time, of two of the most distinguished of the world's mathematicians, attracted much attention throughout the country. Professor Cayley was present at a number of university receptions given in his honor, and was also widely entertained in a social way. His portrait at the university was placed in a prominent position yesterday when the news of his death was received, and appropriately draped with black.—F. P. MATZ.

P. L. Tchebichef, member of the St. Petersburg Academy of Science, died Dec. 8th last, at the age of 74. Dr. Halsted has written a biography of this great mathematician, which will appear in the March number of the MONTHLY.

On February 1, G. H. Harvill, Tyler, Texas, will begin the publication of a Monthly School Journal, "*The Investigator.*" This new journal is to be devoted to Elementary Mathematics, English Grammar, History, Geography, Philosophy, etc. The price of the Journal will be $1.00 per year. The editors of the MONTHLY extend a hand of welcome to this new periodical and wish it abundant success.

Dr. George Bruce Halsted has just received a letter from Dairoku Kikuchi, Member of the House of Peers of Japan, Professor of mathematics in the Imperial University of Tokyo, Japan, in which he says:

"I desire to acknowledge with many thanks your translation of Professor Vasiliev's Address. The Russian original I received some time ago, but I regret that I could not read it. Your Bolyai ought to have been printed here long ago, but owing to unavoidable circumstances has been delayed. It will be ready, however, in a week from now, and then I shall send you some copies at once.

We are fighting the battle of Civilization against the Chinese, and with easy success which even the most sanguine of us had scarcely expected before the war began."

[It may be remembered that the Japanese issued for their own use, in English, a beautiful edition of Dr. Halsted's Lobachevsky, and asked permission to issue his Bolyai.]

The publication of the weekly journal *Science* was resumed January 1st, 1895, under the charge of an editorial committee, of which Simon Newcomb represents mathematics. Professor J. McKeen Cattell of Columbia College, Editor of the Psychological Review, in asking Dr. Halsted to give an account of an Italian and German work, adds: "You are probably the only eminent man of Science in America who reads Russian, so any report of recent scientific work from Russia would be very acceptable." The MONTHLY is proud to have the illustrious name of Dr. Halsted on its list of contributors.

EDITORIALS.

THIS number of the MONTHLY was mailed Feb. 13. The February number will be issued as soon as possible.

DR. HALSTED'S article on Non-Euclidean Geometry was cut short this month owing to sickness in his family. February MONTHLY will contain the usual number of pages devoted to that wonderful department of mathematics.

THE last two lines of problems 43, pp. 12 and 13, should go to the top of page.

WHAT THEY SAY OF THE MONTHLY.—W. I. Taylor, Instructor in Mathematics, Baldwin University, Berea, Ohio, says: I am so well pleased with the MONTHLY and have received so much help from it, that I feel that I ought not to do without it....Dr. Alexander Macfarlane, Ithaca, N. Y., says: The December number of the MONTHLY has just arrived to-day. I congratulate you upon the success of the first volume. I shall endeavor to give you what assistance I can and I think that the MONTHLY deserves the support of the professors and teachers of mathematics.